Determining the Concentration and Diffusion Velocity of Dark Atoms inside Earth

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Introduction

- Space objects are constantly exposed to flows of cosmic particles of various types and origins. Over the past decades, many ground-based and underground detectors have been built to detect such particles.
- Certain detectors were designed specifically to detect particles that could hypothetically constitute dark matter candidates. Over the years, these detectors have collected a significant amount of observational data (positive or negative), raising questions about verification of existing dark matter models based on these observations.
- The experimental setup obviously influences the way the results are obtained, as well as the potential problems:
  - deep underground detectors should reduce the background and supposed to have sensitivity to weakly interacting DM
  - they might become less sensitive to more strongly interacting DM, which has reduced kinetic energy after repeated scattering against the Earth's crust, depending on the exact way the measurements are performed
  - near-surface direct detection experiments supposed to be more sensitive to strongly interacting DM [Joseph Bramante, Andrew Buchanan, Alan Goodman, and Eesha Lodhi, "Terrestrial and Martian Heat Flow Limits on Dark Matter," Phys. Rev. D 101, 043001 (2020), arXiv:1909.11683 [hep-ph].]
- This gives us the opportunity to test existing models of dark matter, on the other hand, it allows us to explain the results of observations using the very interpretation offered by the models.

Dark Atoms

- Hypothetical stable lepton-like particles with a charge of $-2n$ (where n is any natural number) that form neutral bound states with n primordial helium nuclei.

[Belli, P., Bernabei, R., Beylin, V., Bikbaev, T., Kharakhashyan, A., Khlopov, M., Korchagin, V., Mayorov, A., & Sopin, D. (2026). A Dark Atom Scenario for Direct Dark Matter Investigation. *Universe*, 12(4), 116. https://doi.org/10.3390/universe12040116].

- When primordial helium is formed in Big Bang nucleosynthesis, the excessive $-2n$ charged particles bind to the n nuclei of primordial helium-4 — this is the way to form of Thompson type dark atoms. They are n − α-particle nuclei with electric charge compensated by superheavy $-2n$ charged lepton inside them.

- Dark atoms have strong (nuclear) elastic cross section of interaction with protons and nuclei, however the inelastic process of their radiative capture is strongly suppressed [M. Y. Khlopov, A. G. Mayorov, and E. Y. Soldatov. Composite dark matter and puzzles of dark matter searches. International Journal of Modern Physics D, 19(08n10):1385–1395, 2010.]. Most of restrictions can not be applied to dark atom scenario and describe the point-like particles elastic scattering only.

Dark Atoms

- Dark atoms appear as collisionless gas at the scale of Galaxy. At the average baryonic number density, the Galaxy is also transparent for dark atoms.

$$m_\chi \sim 2 - 10\ TeV$$

- However, baryonic clouds or objects with the size $\sim R$ and baryon number density $n \sim 10^{-4} (1\ \mathrm{TeV}/m_\chi) \mathrm{cm}^{-3}$, satisfying the condition $n\sigma R > 1$ are opaque for dark atoms, and capture them if $n\sigma (m_p/m_\chi) R > 1$.

- The rate of radiative capture of dark atom by nuclei can be estimated with the use of the analogy with the radiative capture of a neutron by a proton:

$$\sigma v = \frac{f\pi\alpha}{m_p^2} \frac{3}{\sqrt{2}} \left(\frac{Z}{A}\right)^2 \frac{T}{\sqrt{A m_p E}}$$

Dark Atom Model Predictions

- The interaction of two bound states of heavy core X^{-2n} should be rare due to the low expected concentration of such particles. There is a possibility of charged bound state overproduction. There should be distinguished negatively charged states (dark ions), neutral particles (dark atoms) and anomalous isotopes with a positive electric charge.
- The reactions should affect the primordial concentrations of ordinary nuclei. The overproduction of primordial metals and anomalous isotopes is expected.

[Belli, P., Bernabei, R., Beylin, V., Bikbaev, T., Kharakhashyan, A., Khlopov, M., Korchagin, V., Mayorov, A., & Sopin, D. (2026). A Dark Atom Scenario for Direct Dark Matter Investigation. *Universe*, 12(4), 116. https://doi.org/10.3390/universe12040116].

- If dark matter interacts strongly enough with nucleons, it will scatter off elements within celestial bodies. If enough scattering occurs, it should thermalize. If DM particles slows down sufficiently, they might remain bound to the object, affecting the mass estimates.
- Strongly interacting DM can could raise the temperature celestial objects through processes like capture and annihilation.

- The probabilities and the impact of such phenomena must be estimated, but this requires thorough assessment of concentrations, rates, velocities, and other factors.
- These estimates should be compared with the observational results.

Chapman-Enskog theory

- Chapman-Enskog theory provides an approach for solving the Boltzmann equations at fixed order in perturbation theory, which allows one to describe the behavior of gases and evaluate their key characteristics.
- It bridges the gap between microscopic particle dynamics and macroscopic fluid mechanics. The Boltzmann equations are modified and the Navier–Stokes–Fourier equations for gases are derived, and it provides formulas to calculate transport coefficients.
- This approach has already been used to describe certain models of dark matter for celestial objects such as the Sun, Earth, Mars, Jupiter, and others.
- Within the framework of Chapman-Enskog theory, the process of collision of dark matter particles with cosmic bodies, or the process of their capture, can be considered as a process of diffusion of two (or possibly more) arbitrary gases.

$$\frac{df}{dt} = \left(\frac{\partial f}{\partial t}\right)_{\text{force}} + \left(\frac{\partial f}{\partial t}\right)_{\text{diff}} + \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

$$f = f^{(0)} + \mathrm{Kn} f^{(1)} + \mathrm{Kn}^2 f^{(2)} + \dots$$

Chapman-Enskog theory modifications

- We partially use a modified framework proposed in [Rebecca K. Leane, Juri Smirnov. Floating Dark Matter in Celestial Bodies. Journal of Cosmology and Astroparticle Physics, V. 2023, N.10, P.57, (2023). arXiv:2209.09834v2 [hep-ph]] for those points that are relevant to the problem and for which we have more or less convincing estimates.
- The original paper proposes a set of approximations that allow one to estimate the number density and diffusion velocity for strongly interacting dark matter particles in the interior of arbitrary celestial bodies. The calculations are carried out taking into account the constant influx of particles from space, their accumulation over time, the loss of particles due to scattering and thermal evaporation, annihilation; thermal and gravitational diffusion.
- We assume that the DM particles are dark atoms propagating through the solar system and penetrating the Earth during rotation of the solar system around the center of the Galaxy. The DM particles propagating inside the Earth slow down to the thermal velocity in the Earth's crust and diffuse towards its center of Earth. During this process, the bound states of the DM particles may occur and possibly detected.

Capture Rate and Scattering Estimation

Escape velocity:

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

G is the gravitational constant
M is the celestial body mass
R is the celestial body radius

Asymmetry factor for scattering:

$$\langle z \rangle \approx \frac{1}{2}$$

Isotropic scattering is assumed

$$\beta = \frac{4m_\chi m_{SM}}{\left(m_\chi + m_{SM}\right)^2}$$

Velocity after *N* scatterings:

$$v_N = v_{esc}(1 - \langle z \rangle \cdot \beta)^{-N/2}$$

Optical depth:

$$\tau = \frac{3}{2}\frac{\sigma}{\sigma_{sat}} = \frac{3}{2} \cdot \frac{\sigma}{\pi R^2 / N_{nuc.obj}}$$

σ_{sat} is the saturation cross section of DM capture

The probability of a single DM particle to undergo *N* scatters:

$$p_N(\tau) = 2\int_0^1 dy \frac{ye^{-y\tau}(y\tau)^N}{N!}$$

Historical particle capture throughout the object's lifetime

Reflection factor:
Varies depending on specific situation

$$f_{cap} \approx \frac{2}{\sqrt{\pi \cdot N_{scat}}} = \left[\frac{2}{\pi} log\left(1 - \langle z \rangle \cdot \beta\right) / l\,og\left(\frac{v_{esc}}{v_\chi}\right)\right]^{1/2}$$

Capture rate for *N* scatters can be formulated as follows:

$$C_N = f_{cap} \times \pi R^2 p_N(\tau) \frac{\sqrt{6} \cdot \rho_\chi}{3\sqrt{\pi} \cdot m_\chi v_\chi} \times \left[\left(2v_\chi^2 + 3v_{esc}^2\right) - \left(2v_\chi^2 + 3v_N^2\right) e^{\frac{-\left(v_N^2 - v_{esc}^2\right)}{2v_\chi^2}}\right]$$

$$v_\chi \approx 230\ \mathrm{km/s}$$

Total capture rate: $$C = \sum_{N=1}^{\infty} C_N$$

$$\rho_\chi \approx 0.01 \frac{\mathrm{M}_\odot}{pc^3}$$

The total number of accumulated DM particles in a celestial body at time τ_{obj} that equal to its age:

$$N_\chi = C \cdot \tau_{obj}$$

Estimation of concentration taking into account the accumulation and influx of new particles

First-order differential equation can be obtained:

$$\frac{\nabla n_\chi(r)}{n_\chi(r)} + (\kappa+1)\frac{\nabla T}{T} + \frac{m_\chi g}{k_B T} = \frac{\Phi}{n_\chi(r) D_{N_\chi}} \frac{R^2}{r^2}$$

Volume integral to account for historically accumulated particles:

$$4\pi \int_0^R r^2 n_\chi(r) dr = N_\chi$$

Particle velocities have a Gaussian distribution and there is no clear preferred direction.

[Brown, W.R.; Geller, M.J.; Kenyon, S.J.; Diaferio, A. Velocity Dispersion Profile of the Milky Way Halo. arXiv **2010**, arXiv:0910.2242v1.]

$$\Phi = v_\chi \sqrt{\frac{8}{3\pi}} \left[1 + \frac{3}{2}\left(\frac{v_{esc}}{v_\chi}\right)^2\right] \frac{\rho_\chi f_{cap}}{m_\chi}$$

At the same time, many articles consider the effects of gravitational focusing of various natures and scales of the effect.

The solution of this system with respect to n(r) provides the concentration inside a celestial object at fixed time τ_{obj}

$$\begin{cases} \dfrac{\nabla n_\chi(r)}{n_\chi(r)} + (\kappa+1)\dfrac{\nabla T}{T} + \dfrac{m_\chi g}{k_B T} = \dfrac{\Phi}{n_\chi(r) D_{N_\chi}} \dfrac{R^2}{r^2} \\ 4\pi \displaystyle\int_0^R r^2 n_\chi(r) dr = N_\chi . \end{cases},$$

Diffusion velocity calculation

- At a depth of about 100 meters, the flow of particles is thermalized, and the process of diffusion towards the center of the Earth begins.
- The direction of the flow changes, the drift component of motion influences the movement of particles and is directed towards the center of the Earth, causing gravitational diffusion.
- The model provides the average v_{diff} depending on depth r

$$f(p) = \sqrt{\frac{2}{\pi}} \frac{p^2}{a^3} e^{-\frac{p^2}{2a^2}}$$

Maxwell–Boltzmann velocity distribution

$$D_{N\chi} \approx \frac{1}{3}\lambda \cdot v_{thermal}$$

$D_{N\chi}$ is the diffusion coefficient

κ is the thermal diffusion coefficient

λ is the mean free path

$$v_{diff}(r) \approx -D_{N\chi}\left(\frac{\nabla n_\chi(r)}{n_\chi(r)} + (\kappa + 1)\frac{\nabla T}{T} + \frac{m_\chi g}{k_B T}\right)$$

Geophysical parameters used for calculation

- Pressure, GPa
- Temperature, K
- Acceleration due to gravity, m/s^2
- Mass density, kg/m^3

Temperature and Mass Density data are from:
Joseph Bramante, Andrew Buchanan, Alan Goodman, and Eesha Lodhi, "Terrestrial and Martian Heat Flow Limits on Dark Matter," Phys. Rev. D 101, 043001 (2020), arXiv:1909.11683 [hep-ph].

Pressure and Free Fall Gravity data are from Radial Reference Earth Model (REM1D):
Moulik P. & G. Ekström (2025) Radial Structure of the Earth: (I) Model Concepts and Data, Phys. Earth Planet. Inter., 361, doi:10.1016/j.pepi.2025.107319
Moulik P. & G. Ekström (2025) Radial Structure of the Earth: (II) Model Features and Interpretations, Phys. Earth Planet. Inter., 361, doi:10.1016/j.pepi.2025.107320

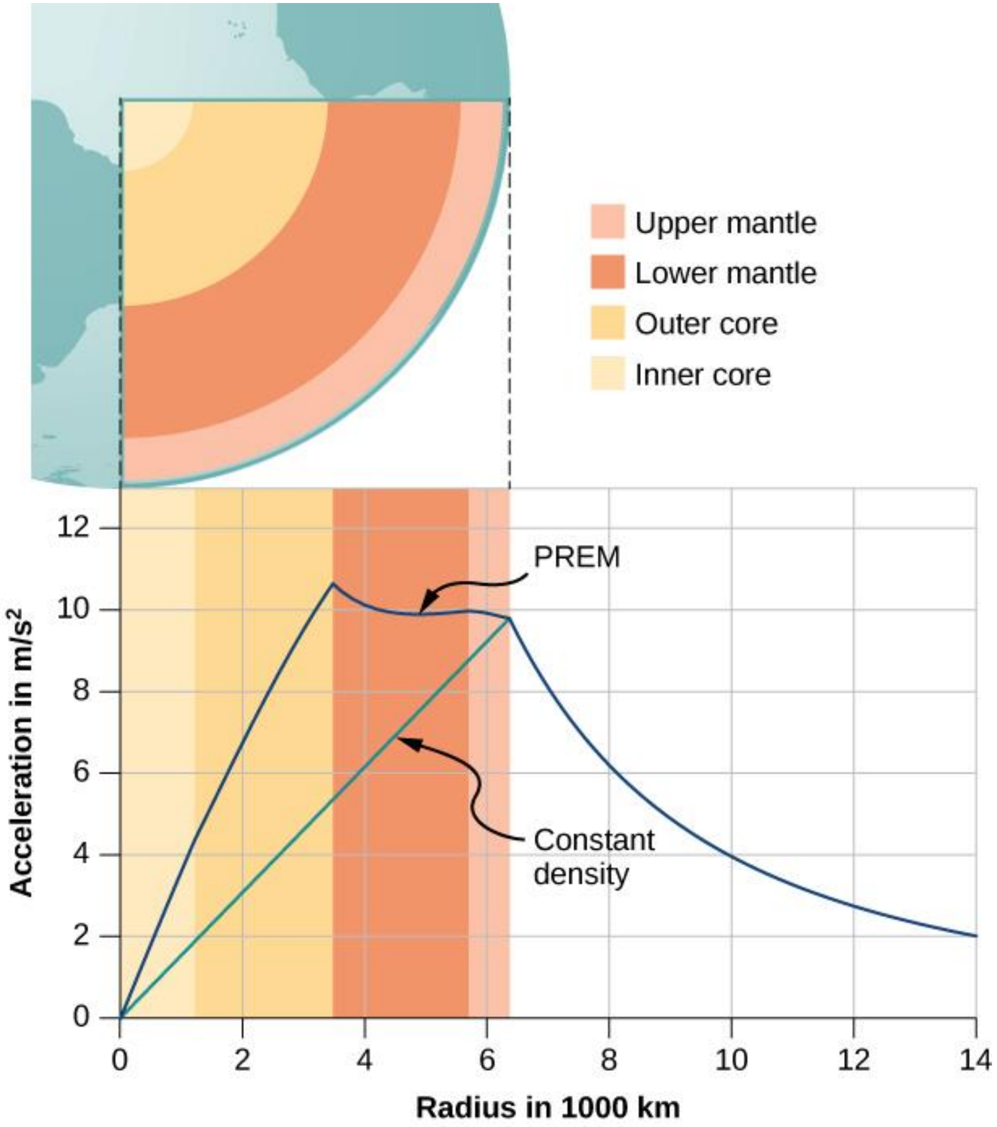
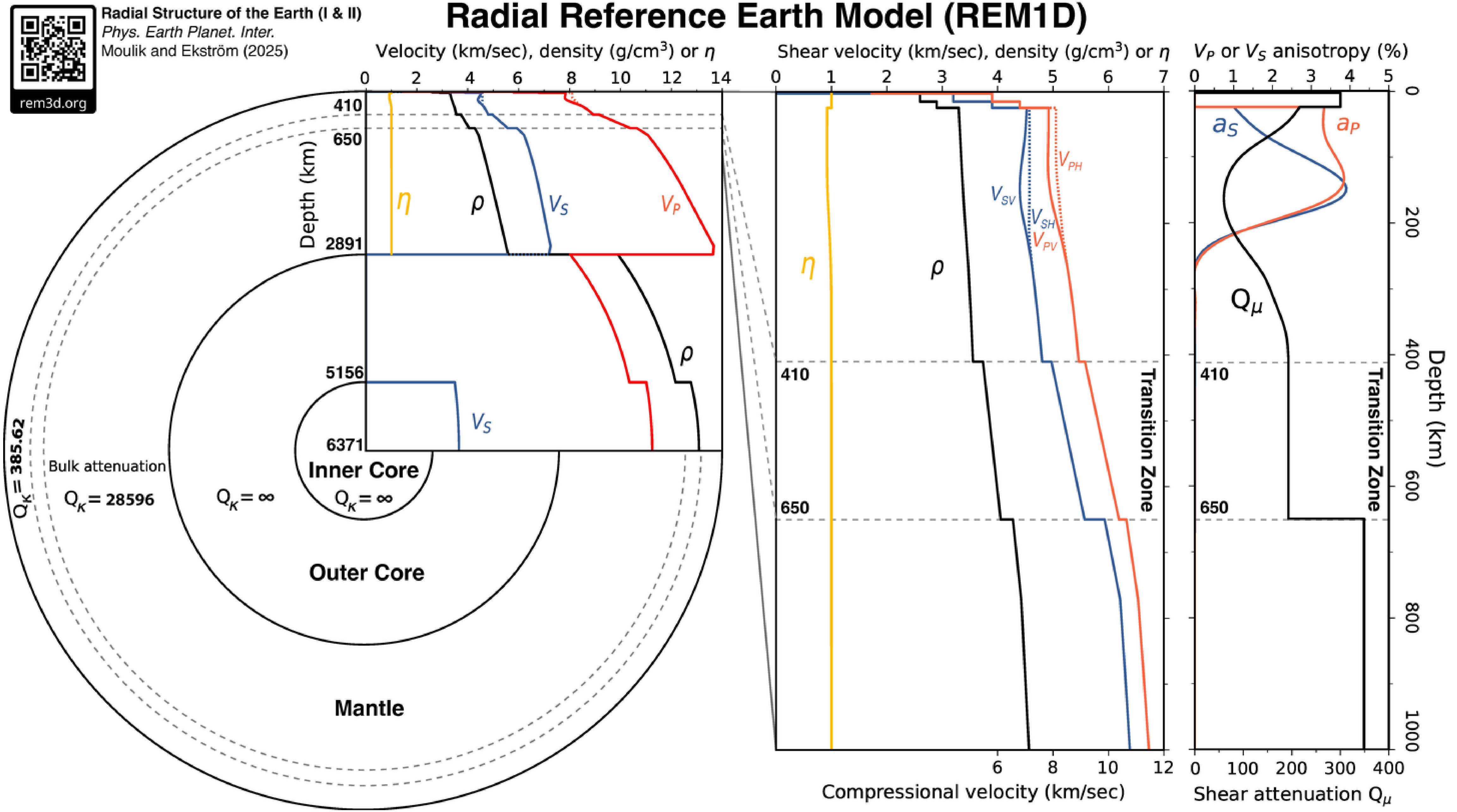


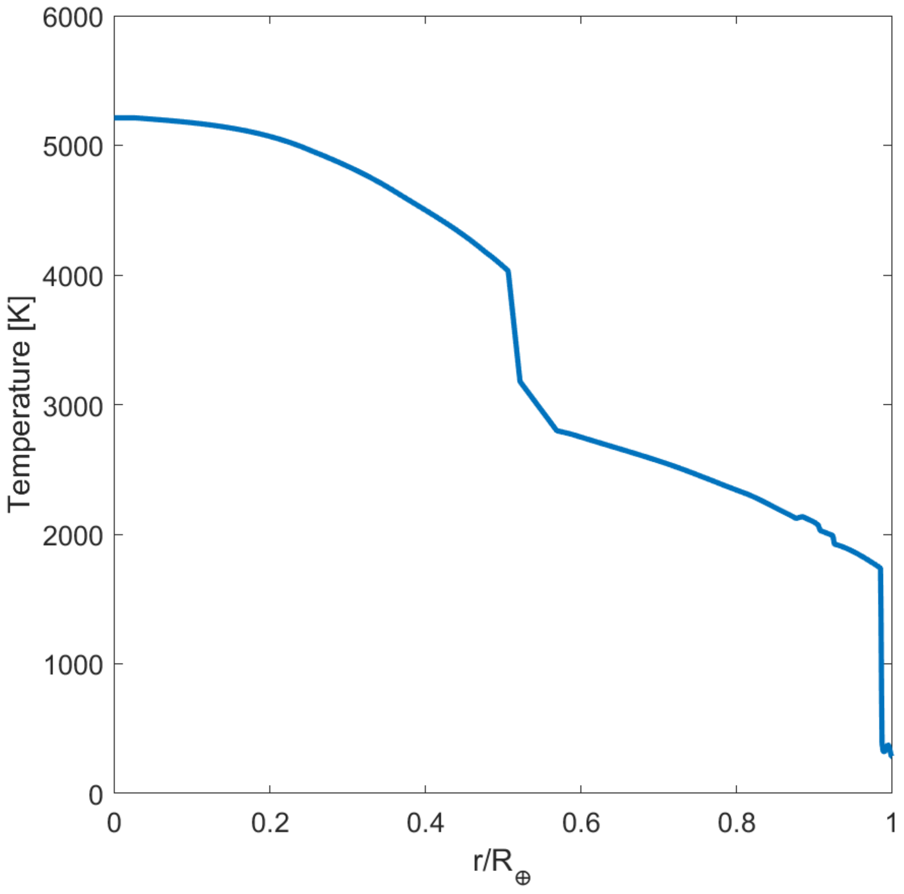
Image from University Physics Volume 1 [https://pressbooks.bccampus.ca/universityphysicssandbox/chapter/gravitation-near-earths-surface/

Radial Reference Earth Model (REM1D)

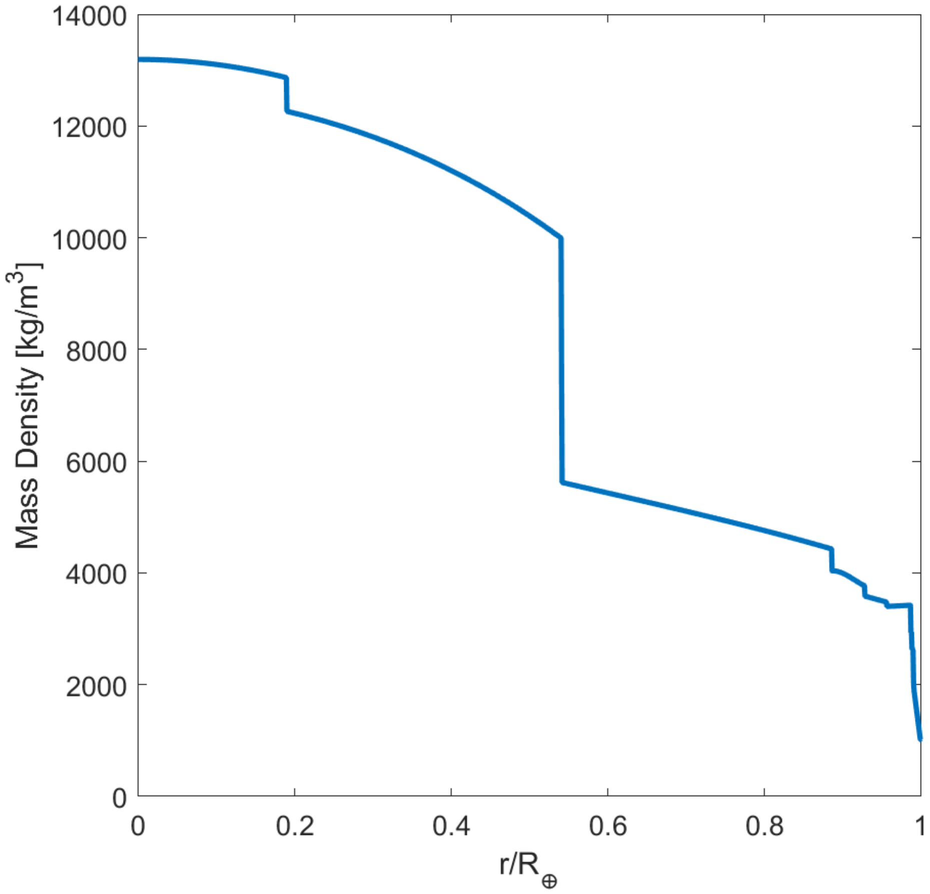


Geophysical parameters profiles
Temperature and Mass Density

Temperature and Mass Density data are from:
Joseph Bramante, Andrew Buchanan, Alan Goodman, and Eesha Lodhi, "Terrestrial and Martian Heat Flow Limits on Dark Matter," Phys. Rev. D 101, 043001 (2020), arXiv:1909.11683 [hep-ph].



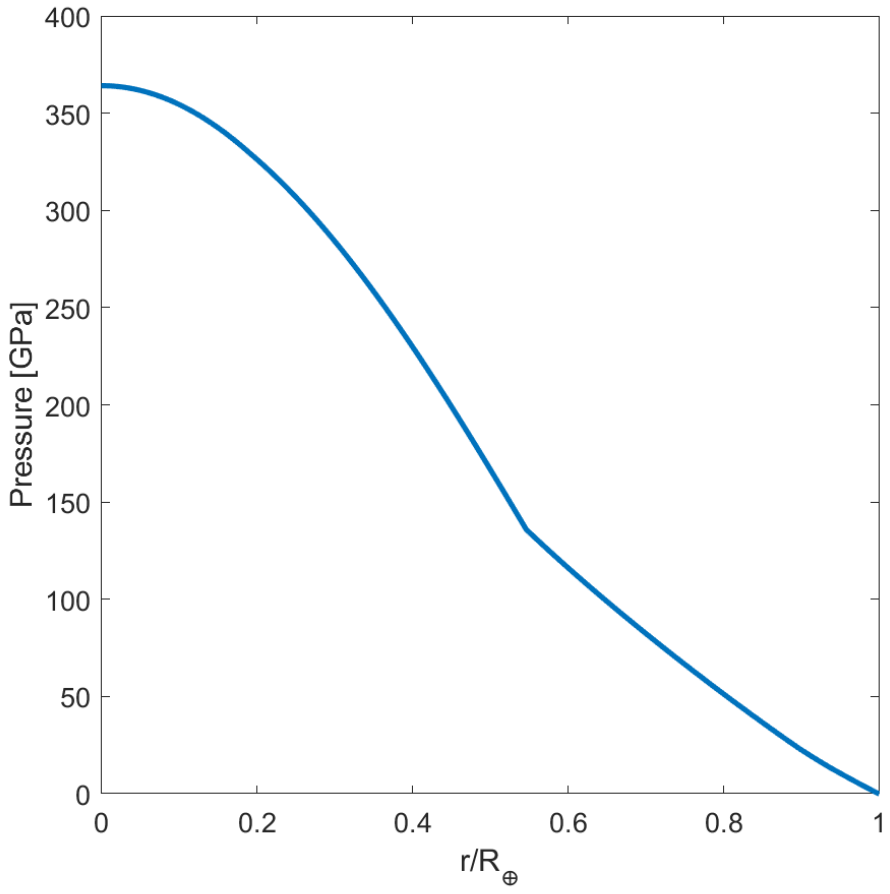
Temperature Profile



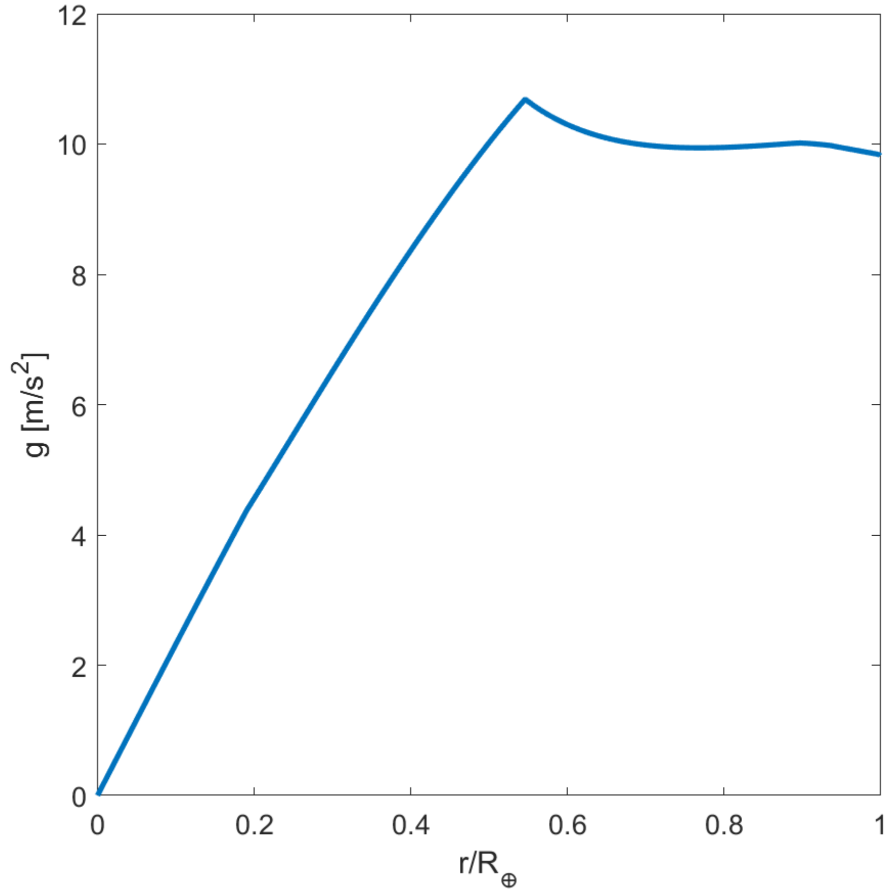
Mass Density Profile

Geophysical parameters profiles
Pressure and Free Fall Acceleration

Pressure and Free Fall Gravity data are from Radial Reference Earth Model (REM1D):

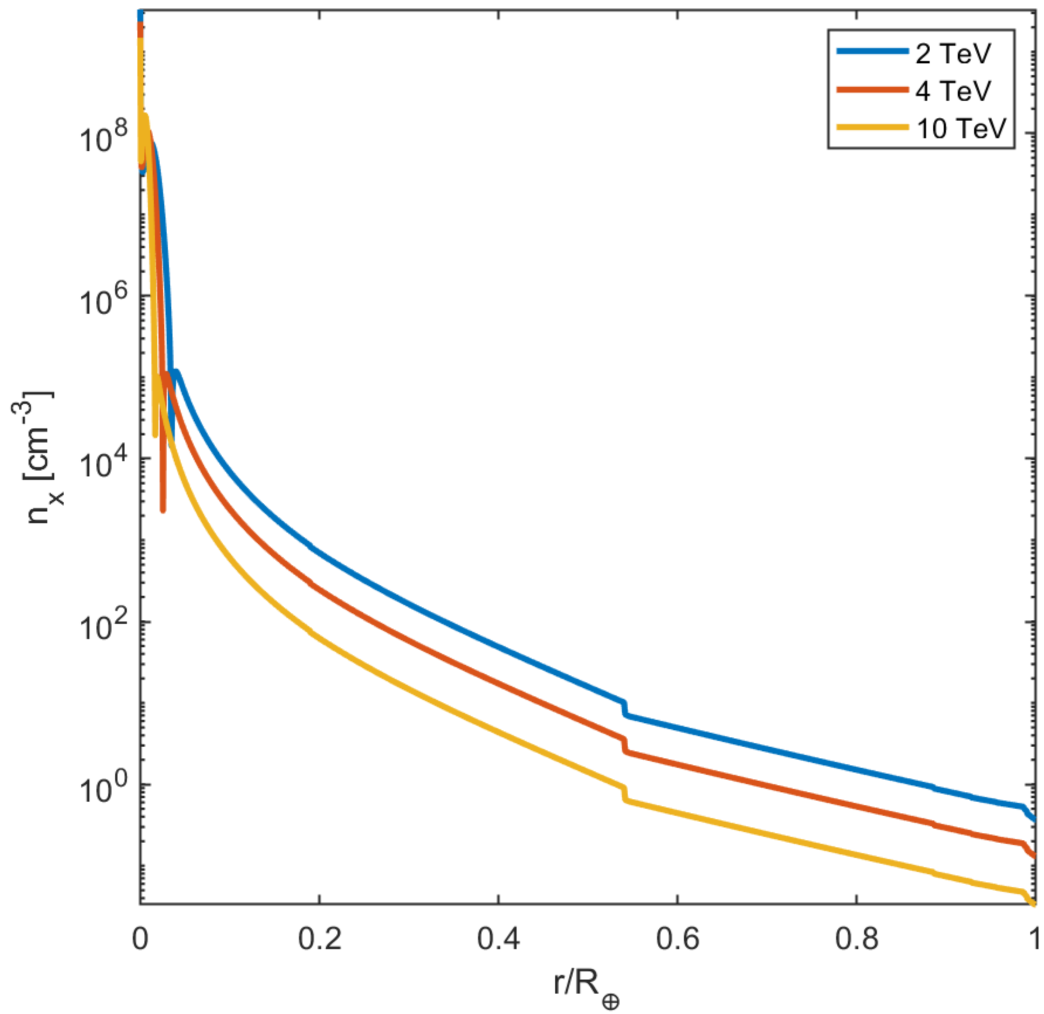


Pressure Profile

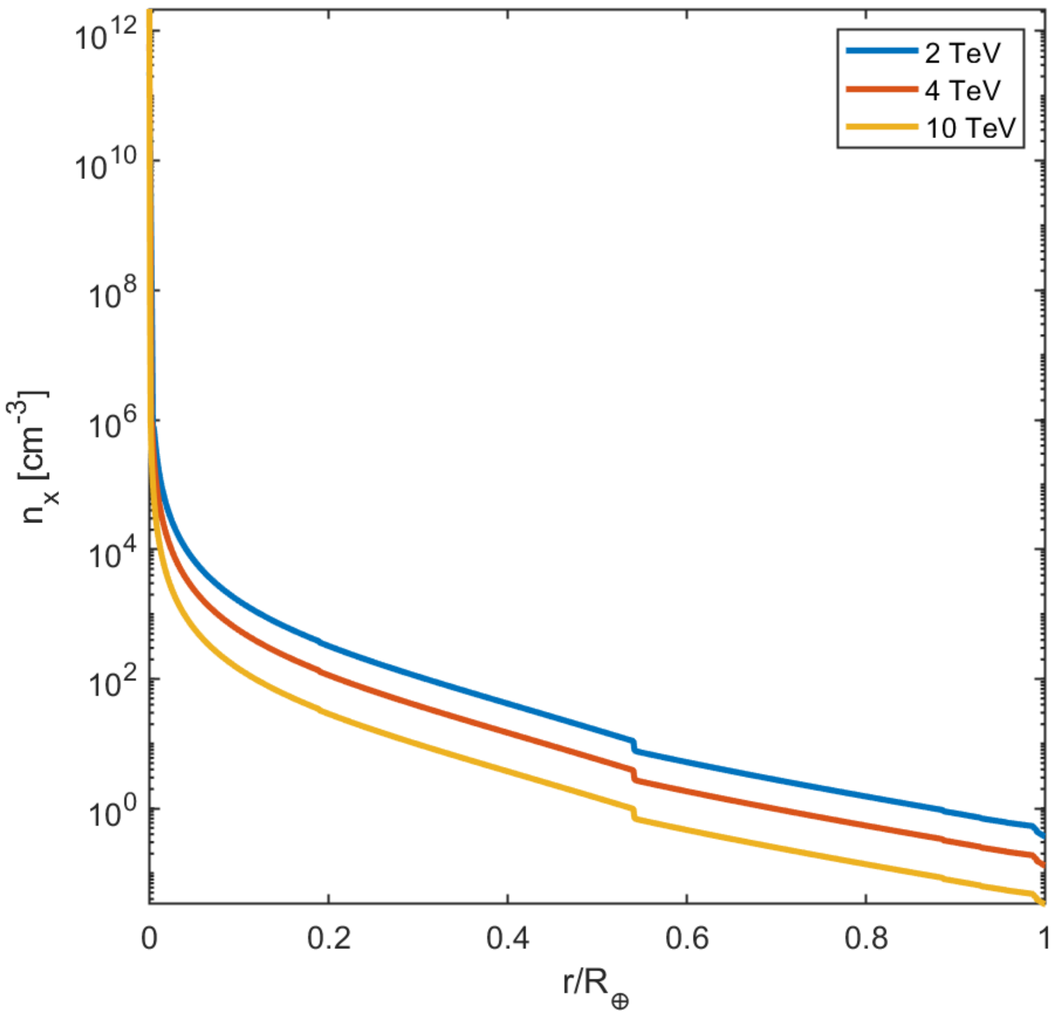


Acceleration due to gravity Profile

Concentration estimates depending on dark atom mass and *g* approximation

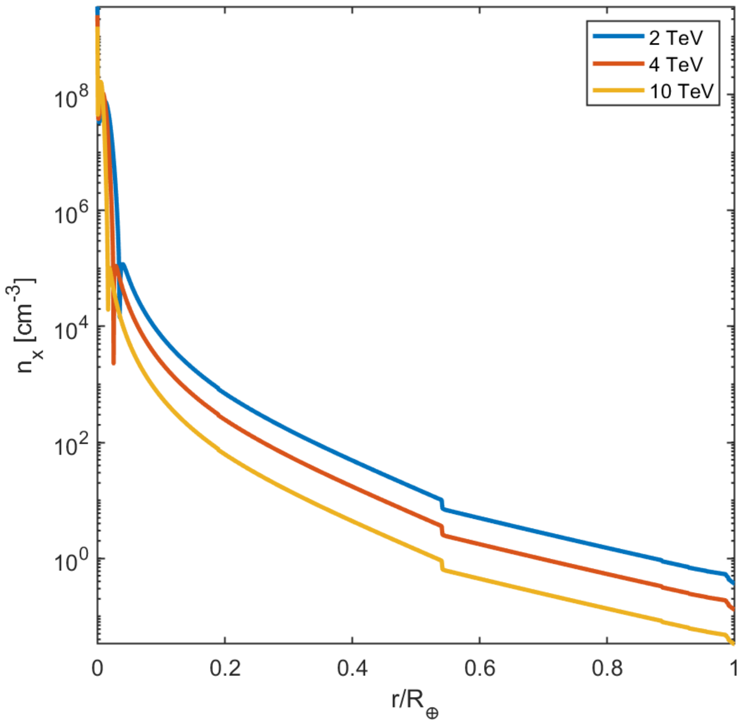


Results under REM1D profile for *g* assumption

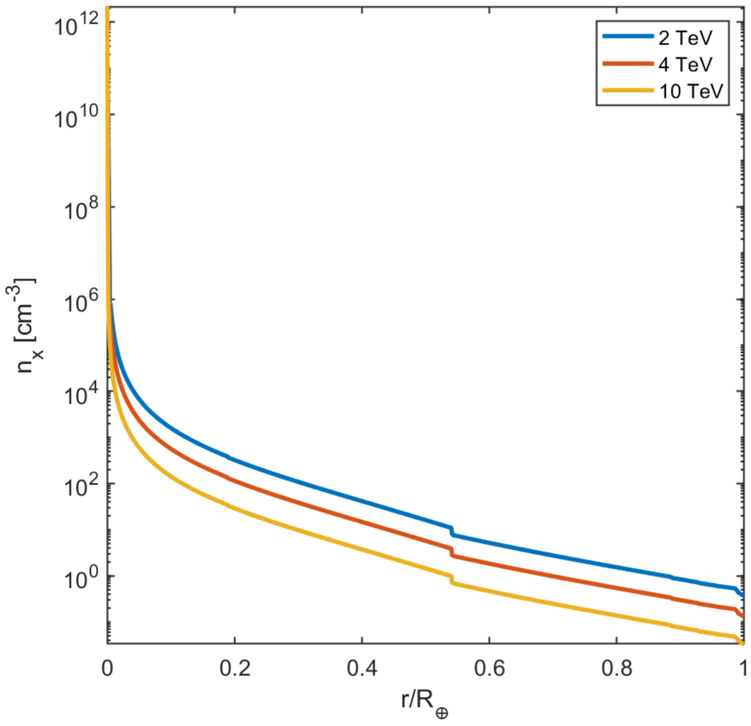


Constant *g* throughout the entire Earth assumption

Comparison with other results



REM1D profile for *g*



Constant *g* throughout the entire Earth

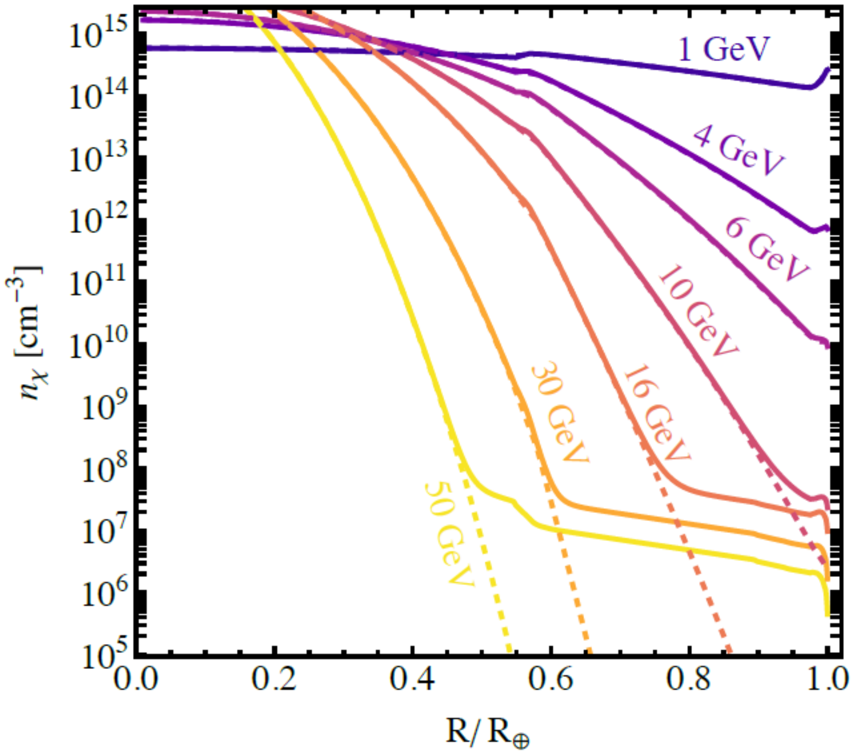


Image from [Rebecca K. Leane, Juri Smirnov. Floating Dark Matter in Celestial Bodies. Journal of Cosmology and Astroparticle Physics, V. 2023, N.10, P.57, (2023). arXiv:2209.09834v2 [hep-ph]]

Results are for much lighter strongly interacting particles of unspecified type

- Numerically $n \sim 0.37\text{-}5.01$ particles / cm^{-3} near the surface, depending on how the incoming particle flux Φ is calculated.
- Results depend on particle mass and *g* approximation
- Temperature and its gradient have a huge impact on the results

Comparison with other results

- Based on [L. P. Lifshitz, E. M.; Pitaevskii. Physical Kinetics, volume 10. Pergamon Press, 1981.], the continuity equation in the case considered can be given as follows:

$$n_0 V_h = n V_{diff}$$

$$v_{diff} = \frac{3T}{m^2 n_{gr} \langle \sigma_t v^3 \rangle} m_\chi g$$

- Here v_{diff} is a velocity of the dark atoms that drift deeply into the Earth under the influence of gravity after thermalization.
- The average mass of ground particles m and their concentration n_{gr} can be estimated for silicates SiO_2, for example.
- The $\langle \sigma_t v^3 \rangle$ is the averaged by the Maxwellian distribution, and the transport cross section can be approximated

$$\sigma_t \sim A^{2/3} \frac{m}{m_\chi} \sigma_{nuclear}$$

The concentration $n \sim 1$ particle / cm^{-3}

- Based on [Khlopov, M. Y., Mayorov, A. G., Soldatov, E. Y. Composite Dark Matter and Puzzles of Dark Matter Searches. arXiv:1003.1144v1 [astro-ph.CO] 4 Mar 2010. https://doi.org/10.1142/S0218271810017962], the average dark atom concentration around the NaI(Tl) detectors of the DAMA experiments can be quantitatively estimated:

$$m_\chi = 2\,TeV \qquad N_T = 4.015 \cdot 10^{24}\ \text{nuclei / kg}$$

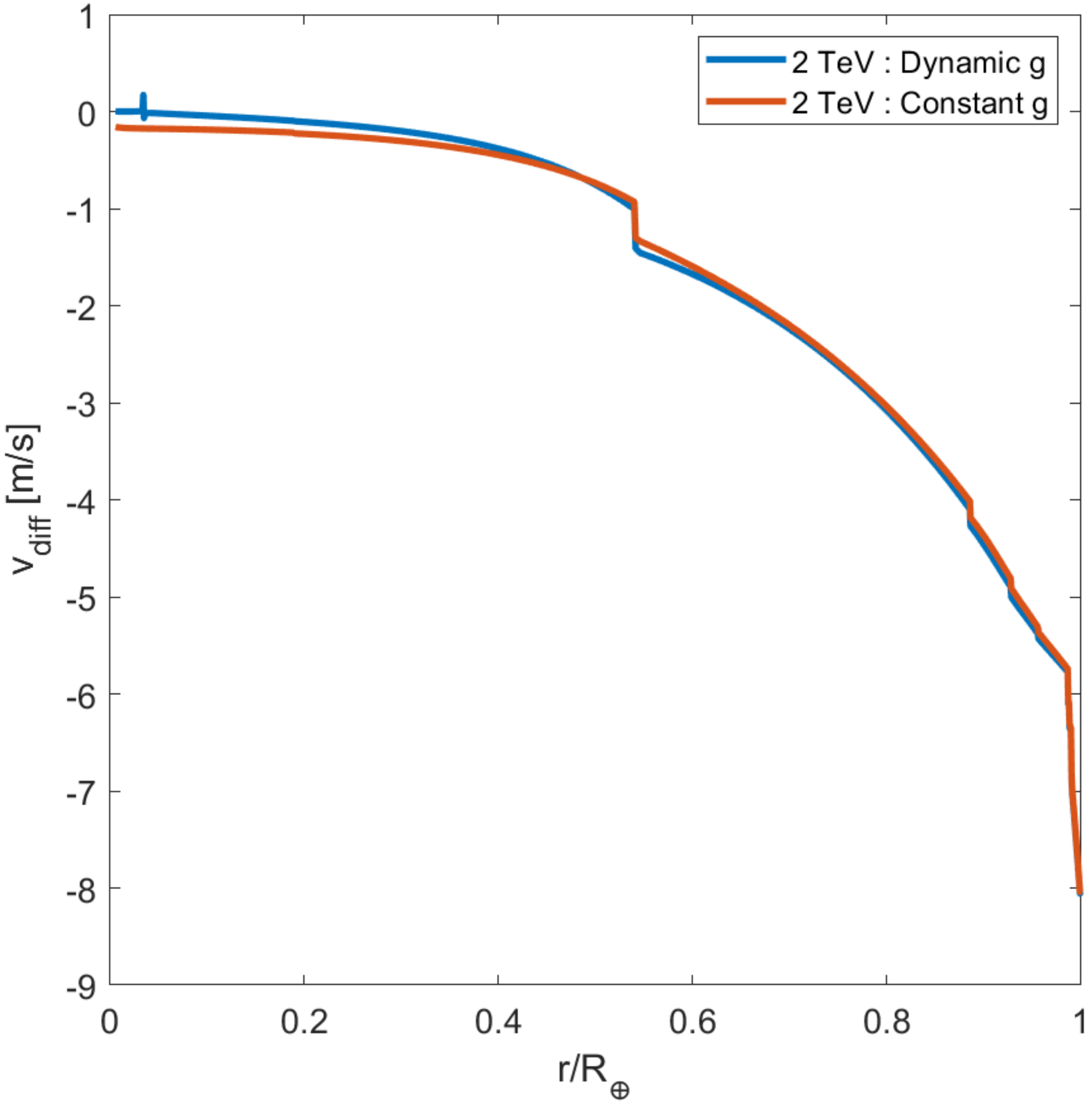
$$\langle \sigma v \rangle_{Na} \approx 4.9 \cdot 10^{-20} \mathrm{MeV}^{-2} = 5.9 \cdot 10^{-31}\ \mathrm{cm}^3/\mathrm{s}$$

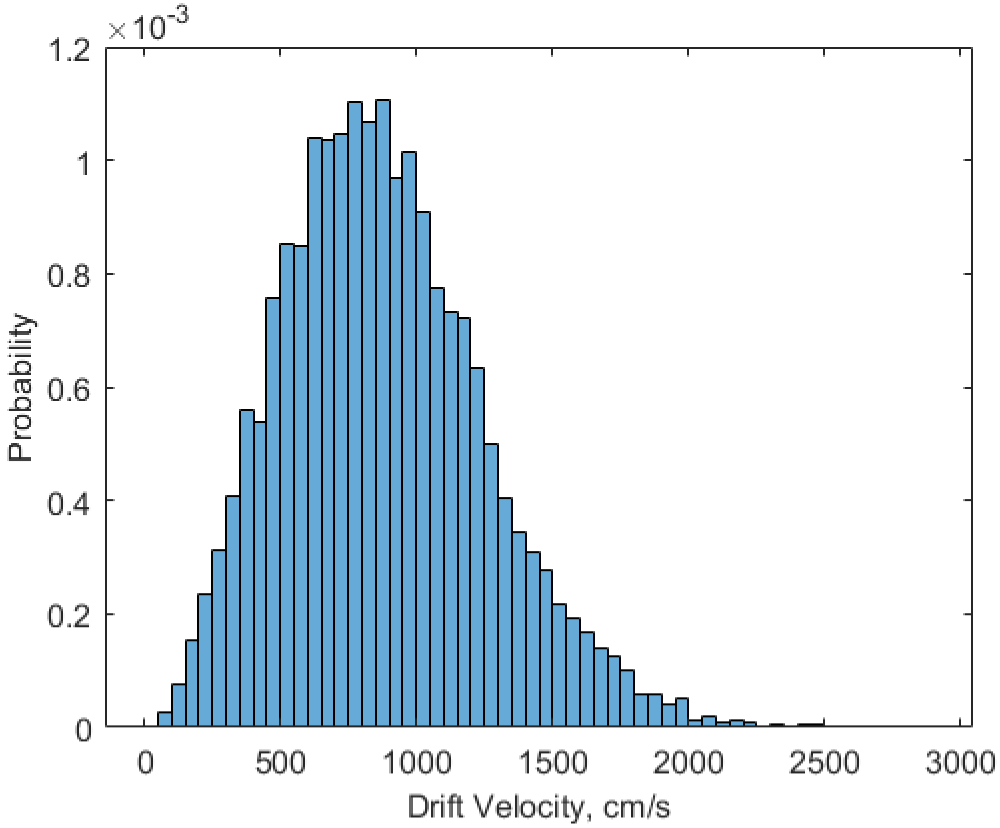
$$\langle \sigma v \rangle_{I} \approx 1.6 \cdot 10^{-20} \mathrm{MeV}^{-2} = 1.9 \cdot 10^{-31}\ \mathrm{cm}^3/\mathrm{s}$$

Count rate: $R = \varepsilon \frac{n(t)}{m_\chi} (\langle \sigma v \rangle_{Na} + \langle \sigma v \rangle_I) N_T$

The concentration $n \sim 2$ particles / cm^{-3}

Diffusion velocity estimates





Diffusion (drift) velocity estimate near the surface from:
Belli, P., Bernabei, R., Beylin, V., Bikbaev, T., Kharakhashyan, A., Khlopov, M., Korchagin, V., Mayorov, A., & Sopin, D. (2026). A Dark Atom Scenario for Direct Dark Matter Investigation. *Universe*, 12(4), 116. https://doi.org/10.3390/universe12040116

$v_{diff} \approx 10^3 \text{ cm/s}$ based on [L. P. Lifshitz, E. M.; Pitaevskii. Physical Kinetics, volume 10. Pergamon Press, 1981.]

General solution for the concentration in 1D

From $$\frac{\nabla n_\chi(r)}{n_\chi(r)} + (\kappa+1)\frac{\nabla T}{T} + \frac{m_\chi g}{k_B T} = \frac{\Phi}{n_\chi(r) D_{N\chi}} \frac{R^2}{r^2}$$ we obtain:

n(r) = C1*exp(-B*r) - exp(-B*r)*(A*B*expint(-B*r) + (A*exp(B*r))/r)

where:

$$expint(x) = \int_x^\infty \frac{e^{-t}}{t} dt$$ — exponential integral (not the Cauchy principal value integral *Ei*)

$$A = \frac{\Phi \cdot R^2}{D_{N\chi}}$$

$$B = (\kappa+1)\frac{\nabla T}{T} + \frac{m_\chi g}{k_B T}$$

Particular solution for the concentration and concentration gradient in 1D

- n(r) = (B*exp(-B*(r - R))*(8*pi*eulergamma*A - B^2*Nx + 8*pi*A*log(R) + 8*pi*A*log(-B) + 8*pi*A*B*R + 8*pi*A*exp(-B*R)*expint(-B*R) + 4*pi*A*B^2*R^2*exp(-B*R)*expint(-B*R) + 8*pi*A*B*R*exp(-B*R)*expint(-B*R)))/(4*pi*(2*B*R - 2*exp(B*R) + B^2*R^2 + 2)) - (A + A*B*r*exp(-B*r)*expint(-B*r))/r

eulergamma is the Euler–Mascheroni constant

$$\gamma = \lim_{n\to\infty}\left(\left(\sum_{k=1}^{n}\frac{1}{k}\right) - \ln(n)\right)$$

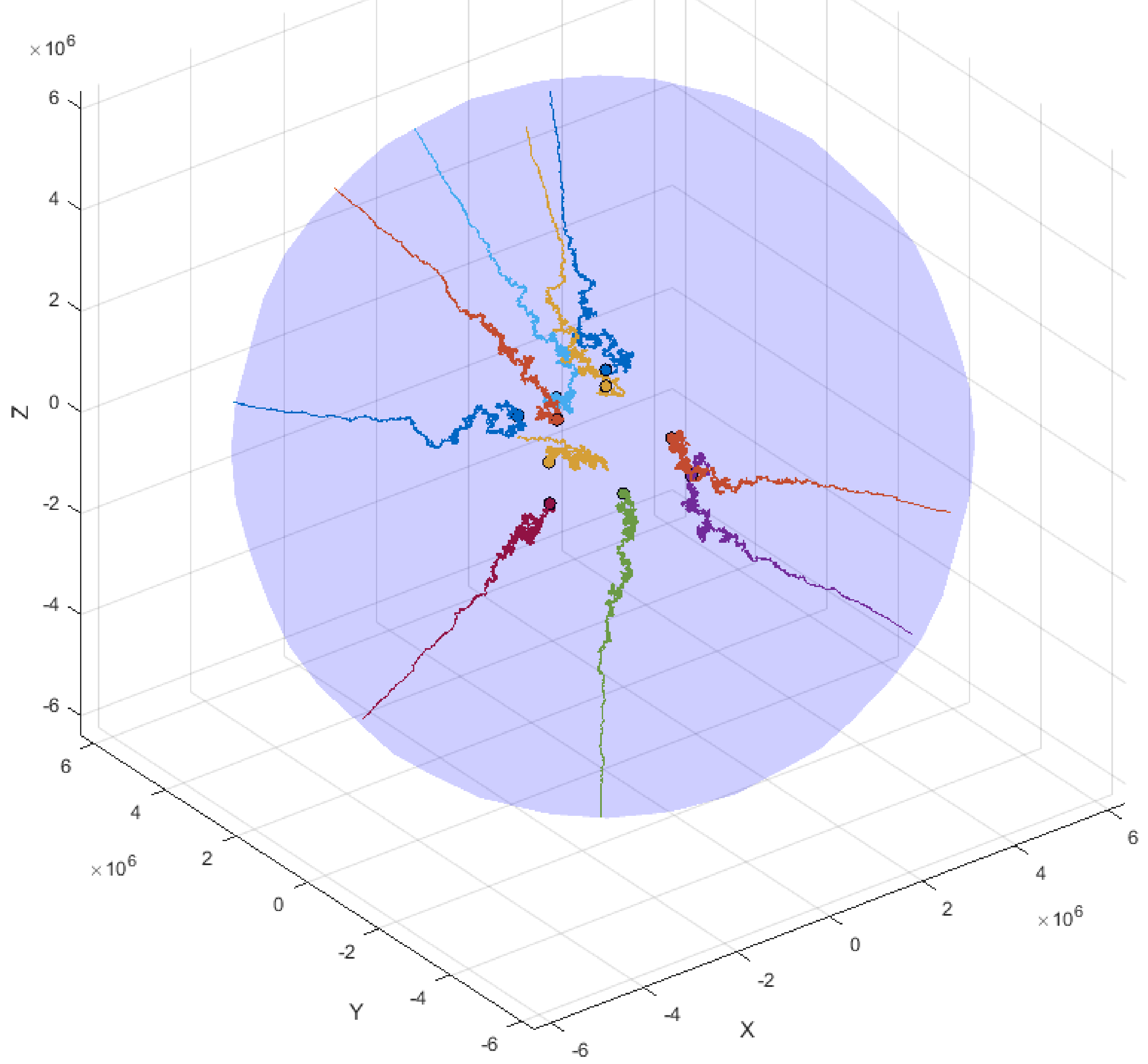
$$expint(x) = \int_{x}^{\infty}\frac{e^{-t}}{t}dt$$

- ∇n(r) = (A*B - A*B*exp(-B*r)*expint(-B*r) + A*B^2*r*exp(-B*r)*expint(-B*r))/r + (A + A*B*r*exp(-B*r)*expint(-B*r))/r^2 - (B^2*exp(-B*(r - R))*(8*pi*eulergamma*A - B^2*Nx + 8*pi*A*log(R) + 8*pi*A*log(-B) + 8*pi*A*B*R + 8*pi*A*exp(-B*R)*expint(-B*R) + 4*pi*A*B^2*R^2*exp(-B*R)*expint(-B*R) + 8*pi*A*B*R*exp(-B*R)*expint(-B*R)))/(4*pi*(2*B*R - 2*exp(B*R) + B^2*R^2 + 2))

$$A = \frac{\Phi \cdot R^2}{D_{N_\chi}}$$

$$B = (\kappa + 1)\frac{\nabla T}{T} + \frac{m_\chi g}{k_B T}$$

Large Scale Brownian Motion with Diffusion Simulation



The idea is to test whether a similar distribution can be obtained using random process theory.

If so, could this approach be generalized to other celestial bodies, where physical parameters are determined with much less certainty than on Earth?

This example is for:

- Exact Scale
- Velocity curves are from the modified Chapman-Enskog model
- 3 spatial dimensions with time
- 10 simulated particles
- 20 millions steps per particle

Simulated Diffusion and Velocity Distribution

- For a set of *p* particles, motion can be described using a two-component Brownian Motion model, where one component describes the drift or diffusion of particles toward the center of the Earth under the influence of gravity, and the other describes the random thermal motion of particles.
- Let us assume that the position of each particle is associated with a radius vector with its origin at the center of the celestial body.
- The particle velocity distribution is given by the Maxwell–Boltzmann distribution with the mean $\approx v_{diff}$ or $\approx v_{thermal}$, depending on the velocity component

$$f(p) = \sqrt{\frac{2}{\pi}\frac{p^2}{a^3}} e^{-\frac{p^2}{2a^2}}$$

- For each particle, the coordinates and velocities are calculated independently, using random sampling from the respective distribution for a given length of the position vector, determining the radial distance.

Brownian Motion with Drift (diffusion)

- Stochastic Tensor Equation

$$\boldsymbol{dBM_t} = -\boldsymbol{v}_{diff} \times \boldsymbol{BM_t} \cdot dt + \boldsymbol{v}_{thermal} \times \boldsymbol{BM_t} \times \boldsymbol{dW_t}$$

- Wiener process (by definition)
  - $W_0 = 0$
  - W_t is almost surely continuous
  - $W_t - W_s \sim \mathcal{N}(0, t - s)$ $\quad 0 \le s \le t$
- Parameters

$\boldsymbol{v}_{diff}$ is the sampled random diffusion velocity vector

$\boldsymbol{BM_t}$ is the coordinate of a particle at time t

$\boldsymbol{v}_{thermal}$ is the sampled random thermal velocity vector

$\boldsymbol{dW_t}$ is the 3D Brownian Motion increments matrix

Limitations and Future Prospects

- Ideal Gas assumption
- Pressure and Mass Density are not included in the solution in a physically correct manner
- Incoming particles flux is approximate and greatly affects the results
- Solution greatly depends on accurate temperature estimates
- Historically accumulated concentration is calculated independently
- Solution is undefined at $r = 0$

- More accurate dark atom properties description
- Better capture rate estimation and boundary conditions
- Modification of the original equations to include more physical parameters, suppression and binding
- Stochastic model with motion equations
- Evaluation for other celestial objects

Conclusions

- In this work, preliminary estimates are obtained for the density profile of dark atom particles in the Earth's interior, taking into account the balance between new particles captured by the Earth, particles accumulated during the object's lifetime, and particles diffusing toward the center of the Earth due to gravitational and thermal diffusion.
- The concentration estimates were used to calculate the diffusion velocity profile of dark atoms inside the Earth. The calculations are performed using the modified Chapman-Enskog theory.
- The obtained results are close to those obtained within the framework of other approaches, and also do not contradict experimental data.
- The obtained diffusion velocity distribution profile is interpretable; the use of a point estimate of the velocity at the surface obtained within the framework of another theory does not lead to anomalous or physically impossible solutions.
- Analytical solutions for the concentration and concentration gradient are obtained for the one-dimensional case.

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1. Joseph Bramante, Andrew Buchanan, Alan Goodman, and Eesha Lodhi, "Terrestrial and Martian Heat Flow Limits on Dark Matter," Phys. Rev. D 101, 043001 (2020), arXiv:1909.11683 [hep-ph].
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THANK YOU FOR YOUR ATTENTION